

# A New Three-Dimensional Rock Mass Strength Criterion

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## Abstract

A new three-dimensional rock mass strength criterion was developed in this paper by extending an existing rock mass strength criterion. This criterion incorporates the effects of the intermediate principal stress, minimum principal stress and the anisotropy resulting from these stresses acting on the fracture system. In addition, the criterion has the capability of capturing the anisotropic and scale dependent behavior of the jointed rock mass strength by incorporating the effect of fracture geometry through the fracture tensor components. The new criterion was proposed after analyzing 284 numerical modeling results of the poly-axial, triaxial and biaxial compression tests conducted on the jointed rock blocks having one or two joint sets by the PFC<sup>3D</sup> software. Some of these simulation results were compared with experimental ones to validate the developed PFC<sup>3D</sup> model that was used for numerical modeling of jointed blocks. In this research to have several samples with the same properties, a synthetic rock material that is made of a mixture of gypsum, sand and water was used. To express the new rock mass strength criterion, it was also necessary to determine the intact rock strengths under the same confining stress combinations mentioned earlier. Therefore, the intact rock was also numerically simulated for all three compression tests and the intact rock strengths were found for 33 different minimum and intermediate principal stress combinations.

## Keywords

Discrete element method (DEM) • Particle flow code (PFC) • Rock mass strength • Polyaxial compression test • Intermediate principal stress • Fracture tensor

## 1 Introduction

Jointed rock masses are known as the combination of intact rock blocks and discontinuities. Due to the presence of complicated discontinuity geometry patterns, the inherent statistical nature of discontinuity geometrical parameters, and the variabilities and uncertainties involved in the estimation of discontinuity mechanical and geometrical properties, estimation of the jointed rock mass strength is difficult and challenging [1]. On the other hand, understanding the rock mass strength is crucial to design safe and economical structures in or on jointed rock masses.

Analytical, Empirical, and numerical are three available approaches to estimate the jointed rock mass strength [1, 2]. Limitations of the analytical and empirical approaches, and advantages of the numerical approaches in estimating rock mass strength are discussed in Kulatilake [2]. Because several parameters affect the strength of rock masses, numerous experimental tests are required to find the effect of these parameters on the strength of rock masses. That task is time consuming, very costly and impractical to perform in the field and laboratory. To solve this problem some researchers modeled rock masses with numerical modeling to propose new rock mass failure criteria. In this method, at first, a numerical model is calibrated with a limited number of experimental tests and physical modeling of the rock masses and then the calibrated model is expanded to more complicated situations with more diverse conditions [3–6]. Kulatilake et al. [1] and Wu and Kulatilake [6] used this procedure incorporating the 3DEC software to find the effect of the joint geometry parameters on the deformability properties of rock masses. To quantify the joint geometry

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parameters, they used an extended form of the fracture tensor concept. Kulatilake et al. [4, 5] and He et al. [3] also extended the fracture tensor concept to fracture tensor components and developed new rock mass strength criteria.

Mehranpour et al. [7] used the same procedure based on experimental tests and PFC<sup>3D</sup> [8] modeling on intact rock, jointed rock with one joint set and jointed rock with two non-orthogonal joint sets to develop new rock mass strength criteria in three dimensions. The new criteria considered the effect of all principal stresses in three dimensions and they are applicable for any type of rock mass, especially for rock masses which generally have non-orthogonal fracture systems. These criteria also show the anisotropic strength behavior of rock masses due to the orientation of joint sets. It should be mentioned that compared to other numerical methods, in the PFC<sup>3D</sup>, macro parameter values are not directly used in the numerical model, and micro parameter values applicable between the particles should be calibrated using the macro property values, and then these calibrated micro parameter values are used in PFC<sup>3D</sup> modeling. This paper presented one of the strength criteria developed in Mehranpour et al. [7] in a summarized form.

## 2 Procedures

In the conducted research to have several samples with the same properties a synthetic rock material that is made of a mixture of gypsum, sand and water was used. To develop a new rock mass strength criterion, first conventional experimental tests on the intact rock and the joint as well as the polyaxial compression tests on the intact rock and jointed samples were performed on the synthetic rock samples. It should be mentioned that polyaxial and triaxial compression tests were performed in the laboratory with a limited number of boundary stress conditions and joint set systems, because the experimental tests are expensive. Then, these experimental tests were simulated using PFC<sup>3D</sup> and the numerical results were compared with the experimental results of synthetic intact rock and synthetic jointed rock blocks. If these two groups of results did not match, micro parameter values of PFC<sup>3D</sup> were modified until very close results were obtained with an acceptable error. Accordingly, the micro properties of the PFC<sup>3D</sup> model were calibrated based on the experimental test results; it turned out to be one of the challenging parts of this project. Afterwards, polyaxial, triaxial and biaxial compression tests for the intact rock and jointed rock blocks were simulated in the PFC<sup>3D</sup> with different combinations of minimum ( $\sigma_3$ ) and intermediate

principal stresses ( $\sigma_2$ ). After gathering the results from 284 numerical simulations, the development of a new rock mass failure criterion was initiated using the fracture tensor concept which was introduced by Oda [9] and developed into the fracture tensor components by Kulatilake et al. [1, 5]. Fracture tensor combines the joint orientation distribution, joint size distribution, joint density for each joint set and the number of joint sets by a second order tensor. Thus, the fracture tensor can show the anisotropy and scale effects of rock masses which are exhibited by the presence of joints. To express the new rock mass strength criterion, it was also necessary to determine the intact rock strengths under the same confining stress combinations mentioned earlier. Therefore, the intact rock was also numerically simulated for all three compression tests and the intact rock strengths were found for 33 different minimum and intermediate principal stress combinations. For details of the experimental and numerical procedures, the reader is referred to Mehranpour and Kulatilake [10] and Mehranpour et al. [7].

## 3 Results

Figures 12–15 of Mehranpour et al. [7] show the rock block strength values obtained for the synthetic jointed and intact rock models under different minimum and intermediate principal stress combinations. These figures indicate that for each combination of the minimum and intermediate principal stresses, the jointed rock blocks with 2 joint sets and 1 joint set have resulted in a lower strength compared to that of the synthetic intact rock and the jointed rock blocks with 2 joint sets have resulted in a lower strength value compared to that of the jointed rock blocks having 1 joint set with the same properties as the first joint set of the rock sample with 2 joint sets. This means that adding of joint sets to a sample under the same minimum and intermediate principal stress combination reduces the strength of the sample. Figure 12 of Mehranpour et al. [7] also shows that the intermediate principal stress has a significant effect on the synthetic intact rock strength and it can increase the intact rock strength up to about 25%. Increase of the intermediate principal stress while keeping the minimum principal stress constant, increases the strength of intact rock to a peak value and then the strength decreases. However, in Figs. 12–15 for each  $\sigma_3$  level in the jointed rock models, the reduction of the strength after reaching the peak strength due to increase of  $\sigma_2$  seems to be lower compared to that of the intact rock model. In some plots, even the strength reduction does not seem to exist especially for low  $\sigma_3$  values and high joint set dip angles.

#### 4 Development of a New Rock Mass Strength Criterion

By subjecting 150 synthetic jointed rock blocks to biaxial loading, Kulatilake et al. [5] developed a biaxial strength criterion for jointed rock masses. Based on extensive laboratory and numerical polyaxial test results on jointed coal blocks, He et al. [3] extended the Kulatilake et al. [5] criterion to the polyaxial compressive stress condition. The obtained results in the current study led to the following observations:

- Increase of joint set dip angles, in general, reduces the jointed rock block strength and increases  $F_{22}$  (the fracture tensor component in  $\sigma_2$  direction) and  $F_{33}$  (the fracture tensor component in  $\sigma_3$  direction). Thus, the increase of  $F_{22}$  and  $F_{33}$  reduces the jointed rock block strength.
- Increase of the minimum and intermediate principal stresses reduces the effect of joint shearing on the jointed rock block strength. Therefore, the increase of the minimum and intermediate principal stresses reduces the effects of  $F_{22}$  and  $F_{33}$ . However, this reduction for low minimum and intermediate principal stresses is relatively higher compared to high minimum and intermediate principal stresses.
- The effect of the minimum principal stress on the joints increases with decreasing angle between the dip direction angle of the joint set and the minimum principal stress direction. Thus, the increase of  $F_{33}$  increases the effect of  $\sigma_3$  on the joints.
- The effect of the intermediate principal stress on the joints increases with the decreasing angle between the dip direction angle of the joint set and the intermediate principal stress direction. Thus, the increase of  $F_{22}$  increases the effect of  $\sigma_2$  on the joints.

Based on the above-mentioned observations the following equation was proposed as a new rock mass strength criterion:

$$S_r = \frac{\sigma_J}{\sigma_I} = \exp - \left[ \frac{\lambda_3}{p_3 \left( \frac{\sigma_3}{\sigma_c} \right)^{q_3} + 1} F_{33} + \frac{\lambda_2}{p_2 \left( \frac{\sigma_2}{\sigma_c} \right)^{q_2} + 1} F_{22} \right] \quad (1)$$

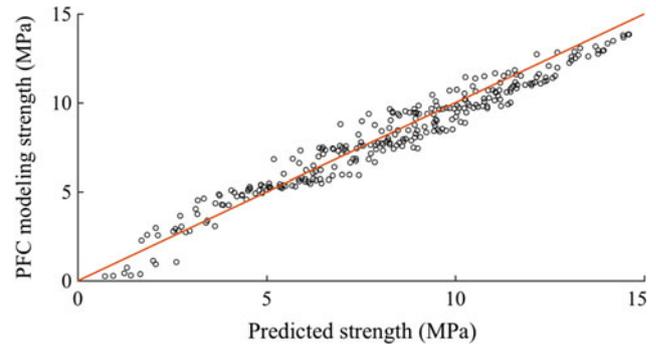
where  $S_r$  is the strength ratio between the jointed rock mass strength,  $\sigma_J$ , under the minimum and intermediate principal stresses  $\sigma_3$  and  $\sigma_2$  and the intact rock strength,  $\sigma_I$ , under the same minimum and intermediate principal stresses,  $\sigma_c$  is the uniaxial compressive strength of the intact rock,  $\lambda_2$ ,  $\lambda_3$ ,  $p_2$ ,  $p_3$ ,  $q_2$  and  $q_3$  are empirical coefficients. It should be

mentioned that if  $\sigma_I$  for the intended  $\sigma_3$  and  $\sigma_2$  combination is not available, based on the Mehranpour and Kulatilake [11] paper one of the three intact rock failure criteria out of the Modified Lade, Modified Wiebols and Cook and Mogi is recommended to represent the intact rock strength value. However, because in this research the intact rock strength for all minimum and intermediate principal stress combinations is available, it is not necessary to use intact rock failure criteria to estimate the intact rock strength.

It should be mentioned that if the joints have the same mechanical properties with isotropic behavior on the joint plane, the effect of  $\sigma_2$  variation on  $F_{22}$  should be the same as the effect of  $\sigma_3$  variation on  $F_{33}$ . Therefore, under this condition Eq. 1 can be simplified to Eq. 2 as follows:

$$S_r = \frac{\sigma_J}{\sigma_I} = \exp - \lambda \left[ \frac{F_{33}}{p \left( \frac{\sigma_3}{\sigma_c} \right)^q + 1} + \frac{F_{22}}{p \left( \frac{\sigma_2}{\sigma_c} \right)^q + 1} \right] \quad (2)$$

In Eq. 2,  $\lambda$ ,  $p$  and  $q$  are empirical coefficients. In the conducted research because all the joints were saw cut, they have the same isotropic mechanical behavior on the joint plane. Thus, to fit the new rock mass strength criterion for the numerical modeling results and to find the accuracy of the new rock mass strength criterion Eq. 2 can be used. The best combination of the empirical coefficients was found by maximizing the coefficient of determination,  $R^2$ . The maximum  $R^2$  was found to be 0.94, indicating a very strong fit. It results in the best values of 0.675, 3.16 and 0.6, for  $\lambda$ ,  $p$  and  $q$ , respectively. Figure 1 shows the predicted strength values versus the strength values from the PFC<sup>3D</sup> modeling for all 284 data points. It indicates that the suggested strength criterion (Eq. 2) is highly suitable to represent the PFC<sup>3D</sup> data.



**Fig. 1** Predicted strength values based on the new rock mass strength criterion based on Eq. 2 versus the strength values from PFC<sup>3D</sup> modeling for all 284 data points from 12 different joint systems having different boundary conditions ( $R^2 = 0.94$ )

## 5 Discussion

The equations given in Sect. 4 to estimate the jointed block strength for synthetic rock are normalized with respect to the synthetic intact rock strength. Therefore, the equations are applicable for any rock mass. The equations allow estimating the normalized jointed block strength in any direction in three dimensions. By estimating the strength in different directions, the strength anisotropy and the minimum normalized jointed block strength can be estimated in three dimensions. The intact block strength can be estimated using one of the available intact rock strength criteria. To estimate the parameters of the intact rock strength criterion, it will be necessary to perform a few laboratory tests as usual. To apply the equations given for normalized jointed block strength for any rock mass, first, the fracture geometry data (number of fracture sets and orientation distribution, size distribution and intensity of each set) should be collected for the intended rock mass. These data will allow the calculation of the two fracture tensor components perpendicular to the direction jointed block strength is desired. These two fracture tensor components go into the normalized jointed block strength equation. The confining stresses should be applied based on the in situ stress system. For the time being the estimated coefficient values of the equations can be used to estimate the jointed block strength. It is important to note that these coefficient values depend on the ratios of joint mechanical property values to intact rock property values. This dependence should be investigated in a future research.

## 6 Conclusion

Based on the observations from synthetic rock testing (on intact, joints and jointed rock) in the laboratory, and the intact and jointed rock modeling results using PFC<sup>3D</sup>, and the fracture tensor component concept, an existing rock mass strength criterion was extended to include the stress anisotropy and develop a new three-dimensional rock mass strength criterion (Eq. 1). The new criterion clearly showed the effect of the intermediate principal stress as well as the

minimum principal stress and joint orientation on the rock mass strength. Because the developed jointed block strength criterion is expressed in a normalized form by dividing by the intact block strength, the normalized jointed block strength criterion is applicable for any rock mass. Guidelines are given to show how the developed strength criterion can be applied to field rock masses.

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